Prep4

October 29, 2015 10:25 AM

Warmup:

$$\frac{1}{x^2-10x-29}+\frac{1}{x^2-10x-45}-\frac{2}{x^2-10x-69}=0$$

Let $y = x^2-10x-29$, then:

$$\frac{1}{y}+\frac{1}{y-16}-\frac{2}{y-40}=0$$

$$\frac{(y-16)(y-40)+y(y-40)-2y(y-16)}{y(y-16)(y-40)}=0$$

Ignore the denominator:

$$(y^2-56y+640)+(y^2-40y)-(2y^2-32y)=0$$

$$-64y+640=0$$

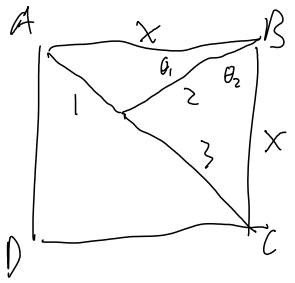
$$y=10=x^2-10x-29$$

$$x^2-10x-39=0$$

$$(x-13)(x+3)=0$$

$$x=13$$

Warmup #2



$$\theta_1=\cos^{-1}\frac{x^2+2^2-1^2}{2\times x\times 2}=\cos^{-1}\frac{x^2+3}{4x}$$

$$\theta_2=\cos^{-1}\frac{x^2+2^2-3^2}{2\times x\times 2}=\cos^{-1}\frac{x^2-5}{4x}$$

$$\theta_1+\theta_2=90^\circ$$

$$\sin\cos^{-1}x=\sqrt{1-x^2}$$

$$\cos(\theta_1+\theta_2)=0$$

$$\cos\theta_1\cos\theta_2-\sin\theta_1\sin\theta_2=0$$

$$\left(\frac{x^2+3}{4x}\right)\left(\frac{x^2-5}{4x}\right)-\sqrt{\left(1-\left(\frac{x^2+3}{4x}\right)^2\right)\left(1-\left(\frac{x^2-5}{4x}\right)^2\right)}=0$$

$$\left(\frac{x^2+3}{4x}\right)^2\left(\frac{x^2-5}{4x}\right)^2 = \left(1-\left(\frac{x^2+3}{4x}\right)^2\right)\left(1-\left(\frac{x^2-5}{4x}\right)^2\right)$$

$$0 = 1-\left(\frac{x^2-5}{4x}\right)^2-\left(\frac{x^2+3}{4x}\right)^2$$

$$\frac{x^4-10x^2+25}{16x^2}+\frac{x^4+6x^2+9}{16x^2}-1=0$$

$$2x^4-20x^2+34=0$$

$$x^4-10x^2+17=0$$

$$x=\sqrt{5+2\sqrt{2}}$$

$$\angle APB = \cos^{-1}\frac{5-x^2}{4}$$

$$= \cos^{-1}\frac{5-(5+2\sqrt{2})}{4}$$

$$= \cos^{-1}\frac{-\sqrt{2}}{2}$$

$$= 135^\circ //$$

Question 1

$$x^6 = y^4+18$$

$$y^6 = x^4+18$$

$$x^6-y^6 = y^4+18-x^4-18$$

$$x^6-y^6 = y^4-x^4$$

$$(x^2-y^2)(x^4+x^2y^2+y^4) = (y^2-x^2)(y^2+x^2)$$

$(x^2-y^2)(x^4+x^2y^2+y^4)+(x^2-y^2)(x^2+y^2)=0$

$(x^2-y^2)(x^4+x^2y^2+y^4+x^2+y^2)=0$

$x^2-y^2=0$

$x^2=y^2$

$x=\pm y$

$x^6=x^4+18$

$x^6-x^4-18=0$

$x^2=z$

$z^3-z^2-18=0$

$z^3-z^2+0z-18=0$

$$\begin{array}{r|cccc} 3 & 1 & -1 & 0 & -18 \\ & & 3 & 6 & 18 \\ \hline & 1 & 2 & 6 & 0 \end{array}$$

$(z-3)(z^2+z+6)=0$

$z=3$

$x^2=3$

$x=\pm\sqrt{3}$

$y=\pm x$

$y=\pm\sqrt{3}$

(x,y) $(\pm\sqrt{3},\pm\sqrt{3})$